Subsurface scattering

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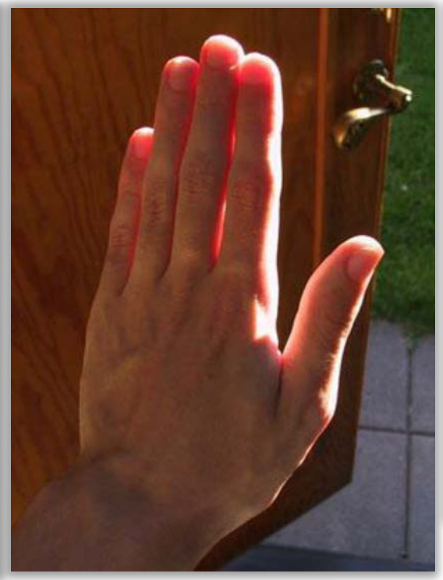
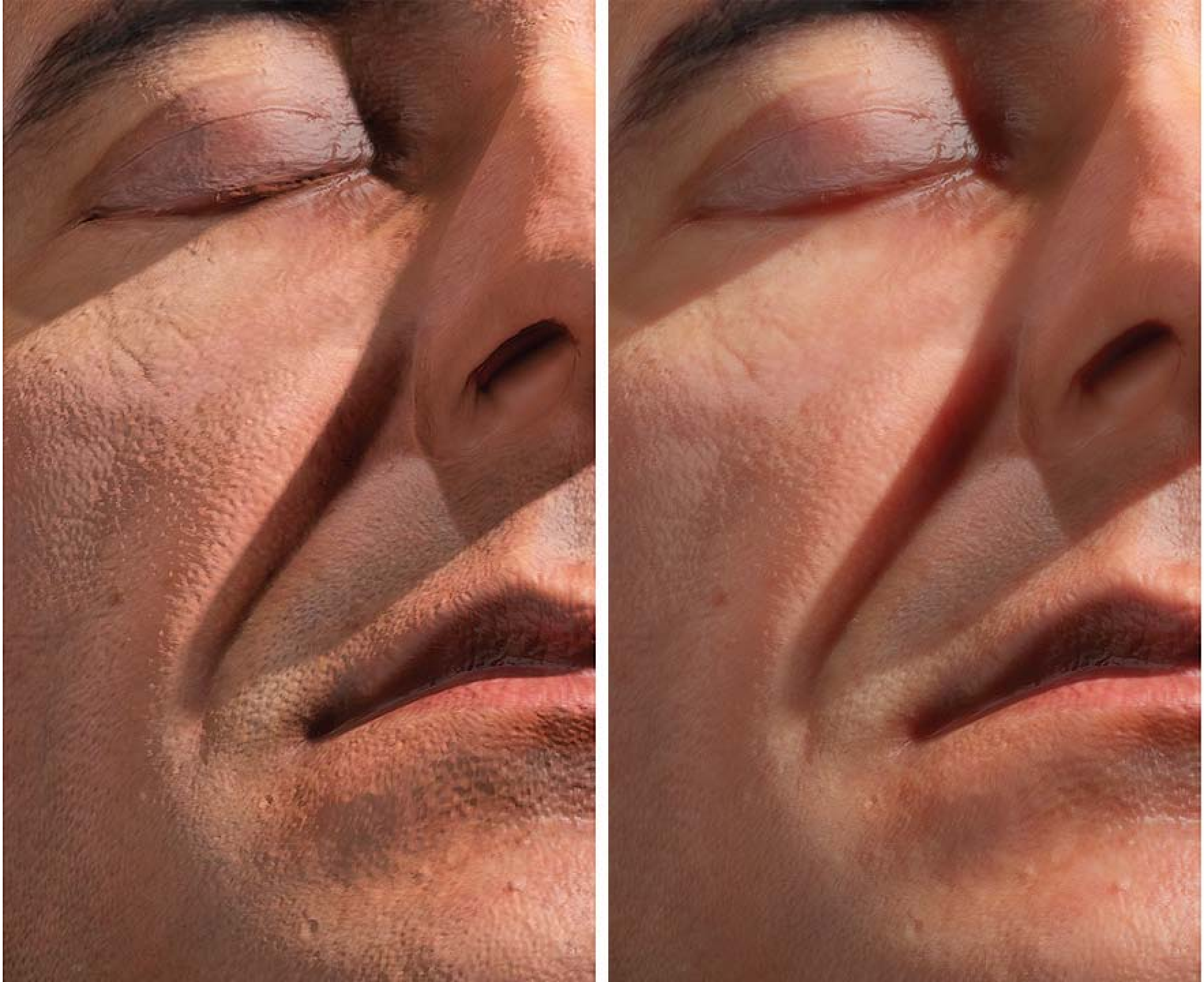
Subsurface scattering examples



Real

Simulated



Video

BSSRDF

- Bidirectional surface scattering distribution function [Nicodemus 1977]
  - 8D function (2x2 DOFs for surface + 2x2 DOFs for dirs)
  - Differential outgoing radiance per differential incident flux (at two possibly different surface points)

$$\boxed{dL_o(x_o,\vec{\omega}_o) = S(x_i,\vec{\omega}_i;x_o,\vec{\omega}_o)\, d\Phi_i(x_i,\vec{\omega}_i)}$$

- Encapsulates all light behavior under the surface

BSSRDF vs. BRDF

- BRDF is a special case of BSSRDF (same entry/exit pt)

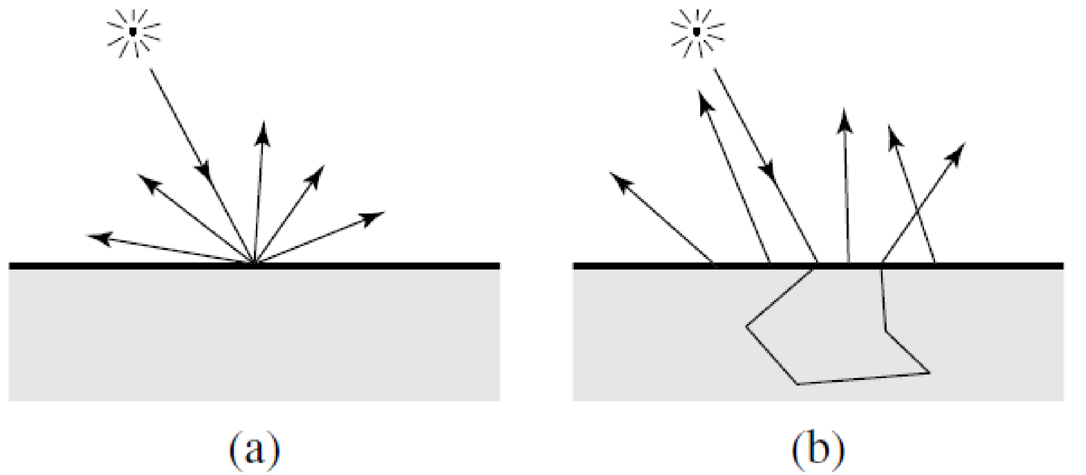


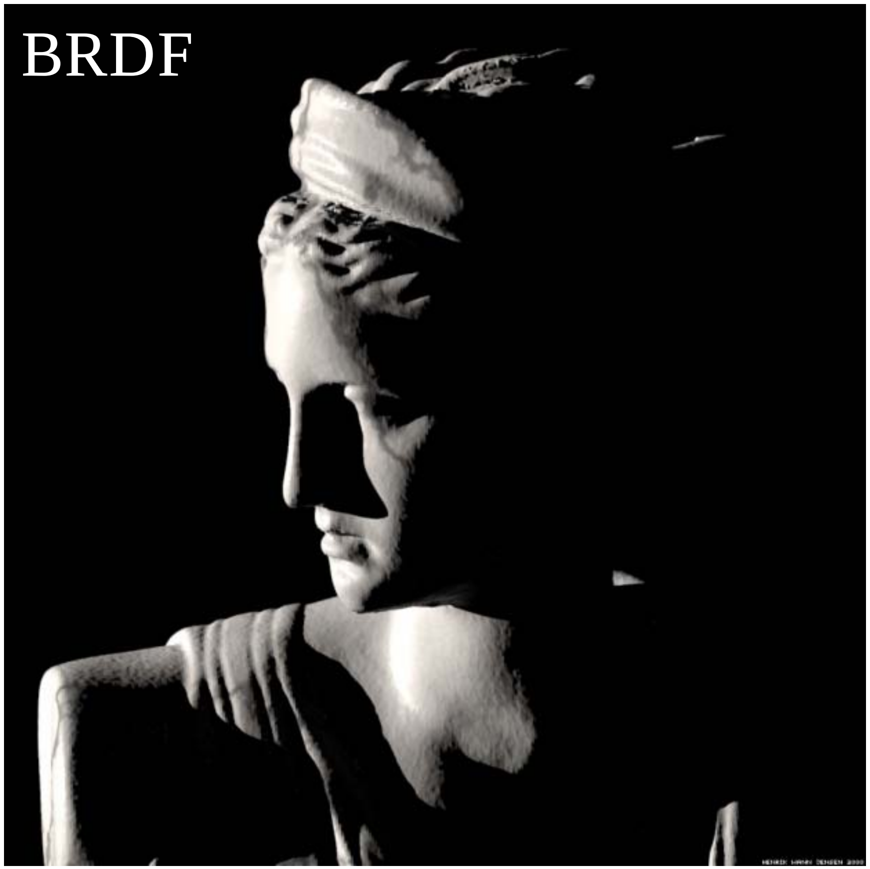
Figure 1: Scattering of light in (a) a BRDF, and (b) a BSSRDF.

BSSRDF vs. BRDF examples 1

BSSRDF vs. BRDF examples

- BRDF – hard, unnatural appearance

BSSRDF vs. BRDF examples



- Show video (SIGGRAPH 2001 Electronic Theater)

BSSRDF vs. BRDF

- Some BRDF model do take subsurface scattering into account (to model diffuse reflection)
  - [Kruger and Hanrahan 1993]
- BRDF assumes light enters and exists at the same point (not that there isn't any subsurface scattering!)

Generalized reflection equation

- Remember that $dL_o(x_o, \vec{\omega}_o) = S(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o)\, d\Phi_i(x_i, \vec{\omega}_i)$

- So total outgoing radiance at x_o in direction ω_o is

$$\boxed{L_o(x_o, \vec{\omega}_o) = \int_A \int_{2\pi} S(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o)\, L_i(x_i, \vec{\omega}_i)\, (\vec{n} \cdot \vec{\omega}_i)\, d\omega_i dA(x_i)}$$

  - (added integration over the surface)

Subsurface scattering simulation

- Path tracing – way too slow
- Photon mapping – practical [Dorsey et al. 1999]

Simulating SS with photon mapping

- Special instance of volume photon mapping [Jensen and Christensen 1998]
- Photons distributed from light sources, stored inside objects as they interact with the medium
- Ray tracing step enters the medium and gather photons

Problems with MC simulation of SS

- MC simulations (path tracing, photon mapping) can get very expensive for high-albedo media (skin, milk)
- High albedo means little energy lost at scattering events
  - Many scattering events need to be simulated (hundreds)

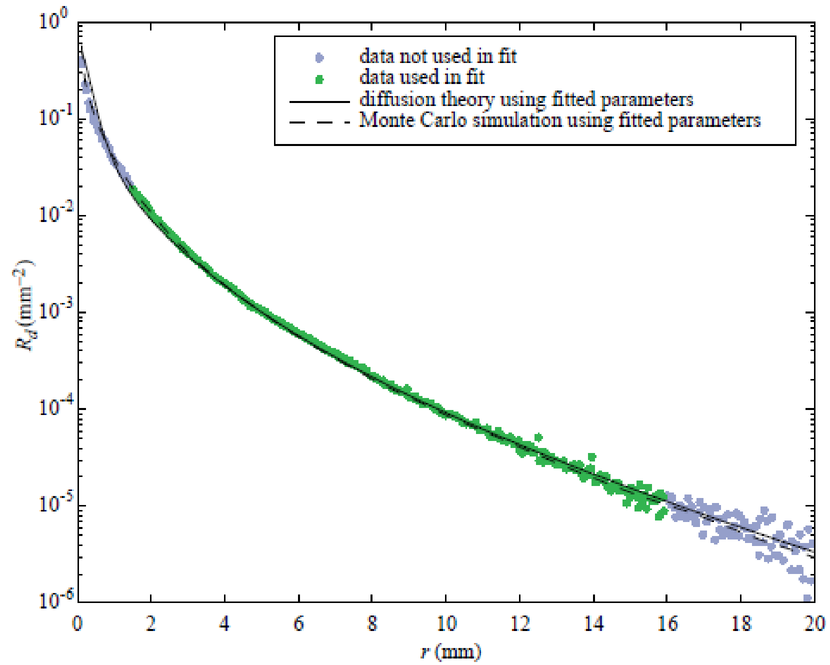
- Example: albedo of skim milk, a = 0.9987
  - After 100 scattering events, 87.5% energy retained
  - After 500 scattering events, 51% energy retained
  - After 1000 scattering events, 26% energy retained

- (compare to surfaces, where after 10 bounces most energy is usually lost)

Practical model for subsurface scattering

- Jensen, Marschner, Levoy, and Hanrahan, 2001
  - Won Academy award (Oscar) for this contribution

- Can find a diffuse BSSRDF $R_d(r)$, where $r = ||\mathbf{x}_0 - \mathbf{x}_i||$
  - 1D instead of 8D !



Practical model for subsurface scattering

- Several **key approximations** that make it possible
  - Principle of similarity
    - Approximate highly scattering, directional medium by **isotropic** medium with modified ("reduced") coefficients
  - Diffusion approximation
    - Multiple scattering can be modeled as **diffusion** (simpler equation than full RTE)
  - Dipole approximation
    - Closed-form solution of diffusion can be obtained by placing two virtual point sources in and outside of the medium

Approx. #1: Principle of similarity

- Observation
  - Even highly anisotropic medium becomes isotropic after many interactions because every scattering blurs light
- **Isotropic approximation**

Approx. #1: Principle of similarity

- Anisotropically scattering medium with **high albedo** approximated as **isotropic** medium with
  - reduced scattering coefficient: $\sigma_s' = \sigma_s(1-g)$
  - reduced extinction coefficient: $\sigma_t' = \sigma_s' + \sigma_a$
  - (absorption coefficient stays the same)

- Recall that *g* is the **mean cosine** of the phase function:

$$g = \int_{4\pi} (\vec{\omega} \cdot \vec{\omega}')p(\vec{\omega} \cdot \vec{\omega}')\, d\omega'$$

  - Equal to the anisotropy parameter for the Henyey-Greenstein phase function

Reduced scattering coefficient

$$\boxed{\sigma'_s = \sigma_s(1-g)}$$

- Strongly forward scattering medium, $g = 1$
  - Actual medium: the light makes a strong forward progress
  - Approximation: small reduced coeff => large distance before light scatters

- Strongly backward scattering medium, $g = -1$
  - Actual medium: light bounces forth and back, not making much progress
  - Approximation: large reduced coeff => small scattering distance

Approx. #2: Diffusion approximation

- We know that radiance mostly isotropic after multiple scattering; assume homogeneous, optically thick
- Approximate radiance at a point with just 4 SH terms:

$$L(x, \vec{\omega}) = \frac{1}{4\pi}\phi(x) + \frac{3}{4\pi}\vec{\omega} \cdot \vec{E}(x)$$

- Constant term: **scalar** irradiance, or **fluence**

$$\boxed{\phi(x) = \int_{4\pi} L(x, \vec{\omega})\, d\omega}$$

- Linear term: **vector irradiance**

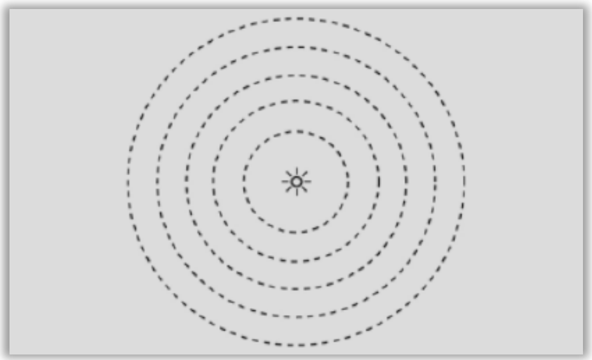
$$\boxed{\vec{E}(x) = \int_{4\pi} L(x, \vec{\omega})\vec{\omega}\, d\omega}$$

Diffusion approximation

- With the assumptions from previous slide, the full RTE can be approximated by the **diffusion equation**
  - Simpler than RTE (we're only solving for the scalar fluence, rather than directional radiance)
  - Skipped here, see [Jensen et al. 2001] for details

Solving diffusion equation

- Can be solved numerically
- Simple analytical solution for point source in infinite homogeneous medium:



source flux

$$\boxed{\phi(x) = \frac{\Phi}{4\pi D} \frac{e^{-\sigma_{tr} r(x)}}{r(x)}}$$

distance to source

- Diffusion coefficient: $D = \frac{1}{3\sigma_t'}$
- Effective transport coefficient: $\sigma_{tr} = \sqrt{3\sigma_a \sigma_t'}$

Solving diffusion equation

- Our medium not infinite, need to enforce **boundary condition**
  - Radiance at boundary going down equal to radiance incident at boundary weighed by Fresnel coeff (accounting for reflection)

- Fulfilled, if $\phi(0,0,2AD) = 0$ (zero fluence at height 2AD)
  - where

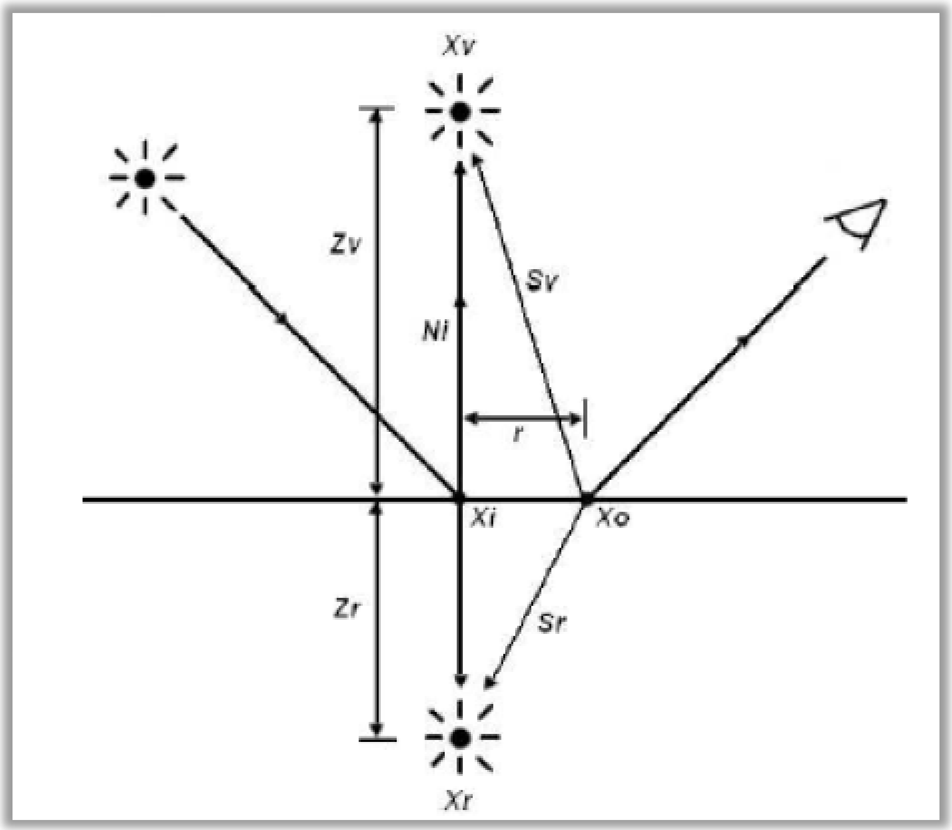
$$A = \frac{1 + F_{dr}}{1 - F_{dr}}$$

  - **Diffuse Fresnel reflectance**

$$F_{dr} = \int_{2\pi} F_r(\eta, \vec{n} \cdot \vec{\omega}')(\vec{n} \cdot \vec{\omega}')\, d\omega' \text{ approx as } -\frac{1.440}{\eta^2} + \frac{0.710}{\eta} + 0.668 + 0.0636\eta$$

Dipole approximation

- Fulfill $\phi(0,0,2AD) = 0$ by placing **two point sources** (positive and negative) inside and above medium



$$z_r = 1/\sigma_t',$$

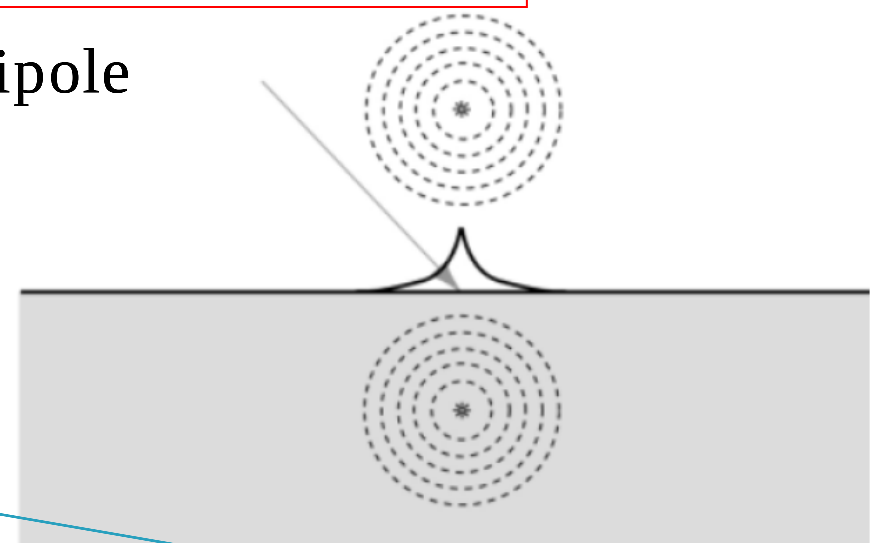
one mean free path below surface

Dipole approximation

- Fluence due to the dipole (d_r … dist to real, d_v .. to virtual)

$$\boxed{\phi(x) = \frac{\Phi}{4\pi\, D}\left(\frac{e^{-\sigma_{tr} d_r}}{d_r} - \frac{e^{-\sigma_{tr} d_v}}{d_v}\right)}$$

- Diffuse reflectance due to dipole



  - We want radiant exitance (radiosity) at surface…
    - (gradient of fluence)
  - … per unit incident flux

$$R_d(r) = -\frac{\vec{\mathbf{N}} \bullet (\nabla \phi_1(r) - \nabla \phi_2(r))}{\Phi_i}$$

Diffuse reflectance due to dipole

- Gradient of fluence per unit incident flux

gradient in the normal
direction = derivative
w.r.t. z-axis

$$R_d(r) = -D\frac{(\vec{n}\cdot\vec{\nabla}\phi(x_s))}{d\Phi_i}$$
$$= \frac{\alpha'}{4\pi}\left[(\sigma_{tr}d_r+1)\frac{e^{-\sigma_{tr}d_r}}{\sigma'_t d_r^3} + z_v(\sigma_{tr}d_v+1)\frac{e^{-\sigma_{tr}d_v}}{\sigma'_t d_v^3}\right]$$

Diffusion profile

- Plot of R_d

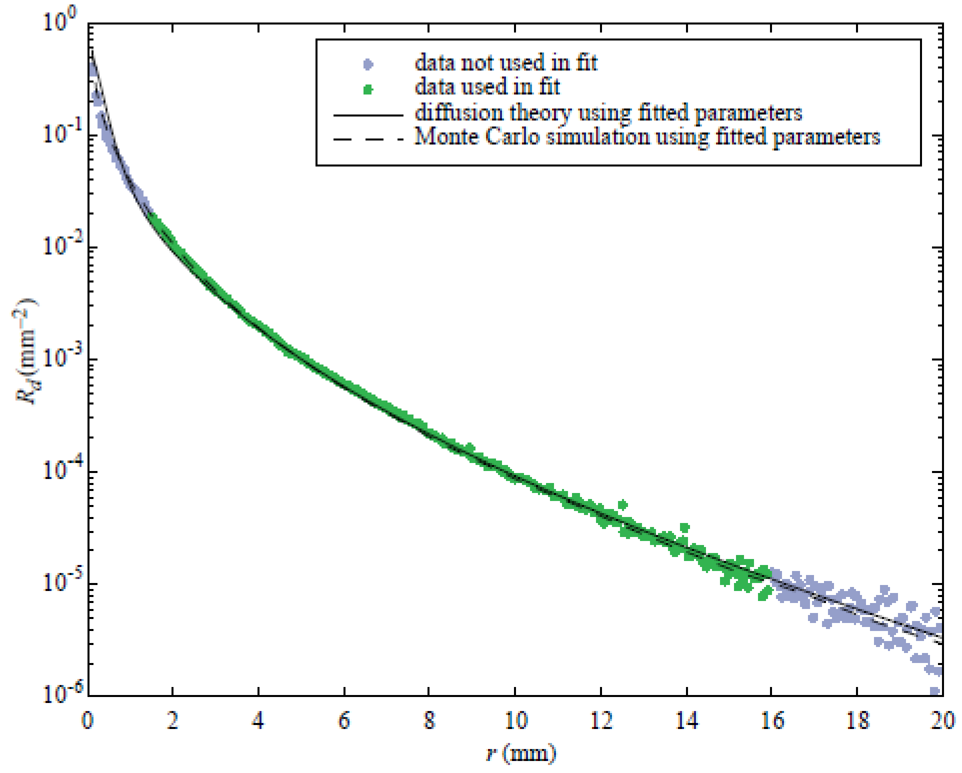


Figure 6: Measurements for marble (green wavelength band) plotted with fit to diffusion theory and confirming Monte Carlo simulation.

Final diffusion BSSRDF

Normalization term
(like for surfaces)

Diffuse multiple-scattering
reflectance

$$S_d(x_i, \vec{\omega}_i; x_o, \vec{\omega}_o) = \frac{1}{\pi} F_t(\eta, \vec{\omega}_i) R_d(||x_i - x_o||) F_t(\eta, \vec{\omega}_o)$$

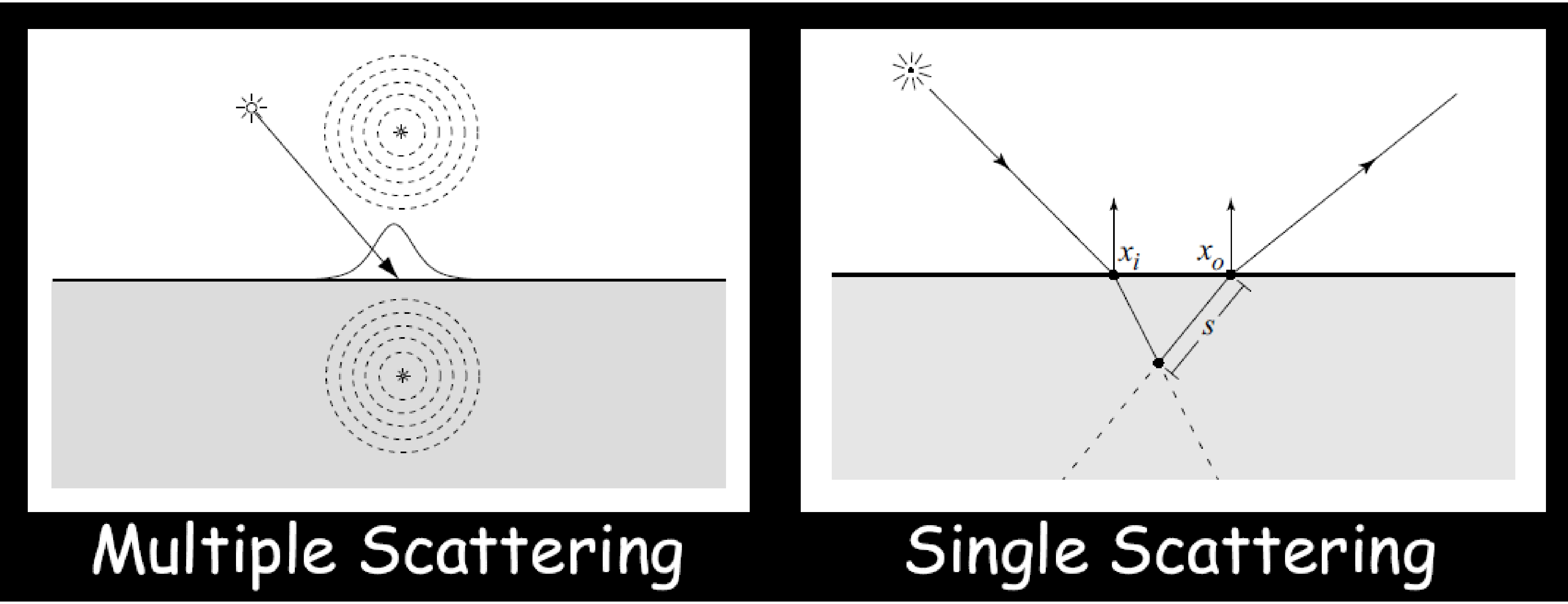
Fresnel term for
incident light

Fresnel term for
outgoing light

Single scattering term

- Cannot be accurately described by diffusion
- Much shorter influence than multiple scattering
- Computed by classical MC techniques (marching along ray, connecting to light source)

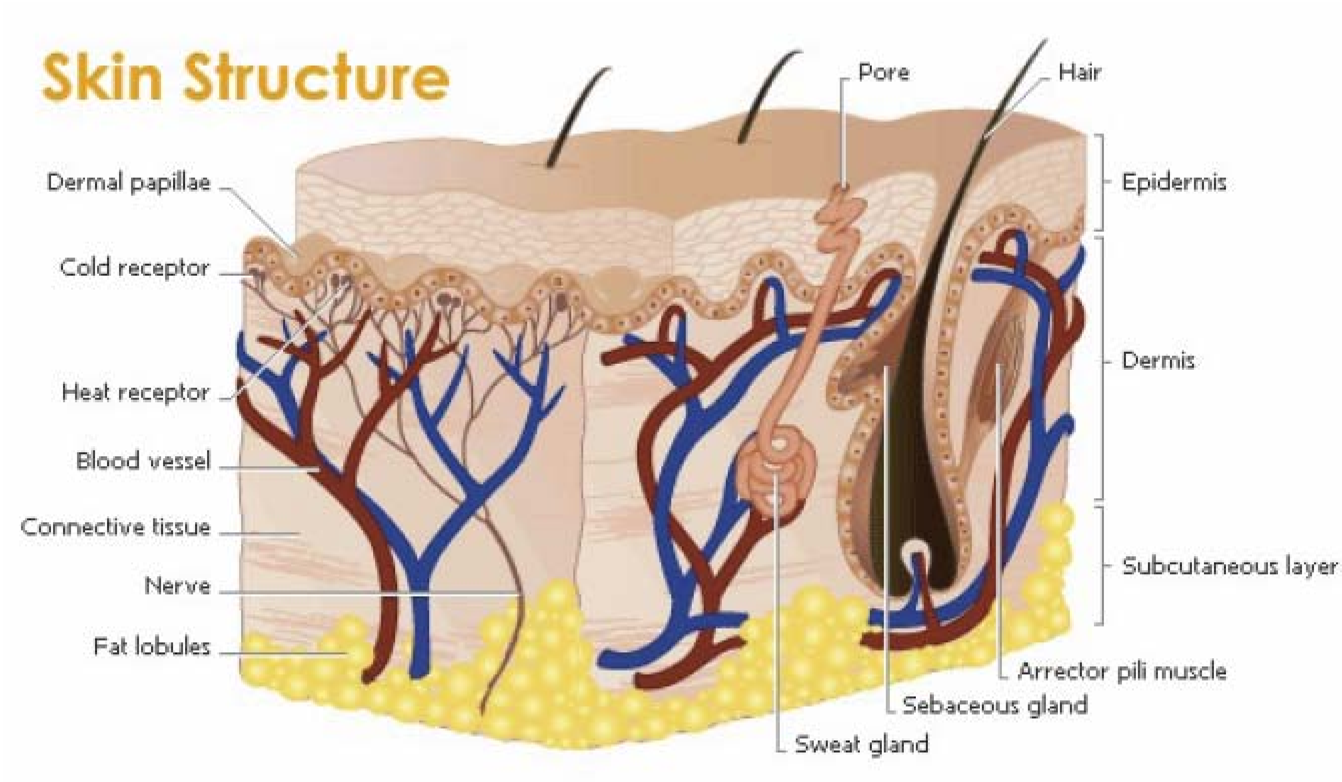
Complete BSSRDF model



MC simulation vs. BSSRDF model

Multiple Dipole Model

- Skin is NOT an semi-infinite slab



Multiple Dipole Model

- [Donner and Jensen 2005]
- Dipole approximation assumed semi-infinite homogeneous medium
- Many materials, namely skin, has multiple layers of different optical properties and thickenss
- Solution: infinitely many point sources

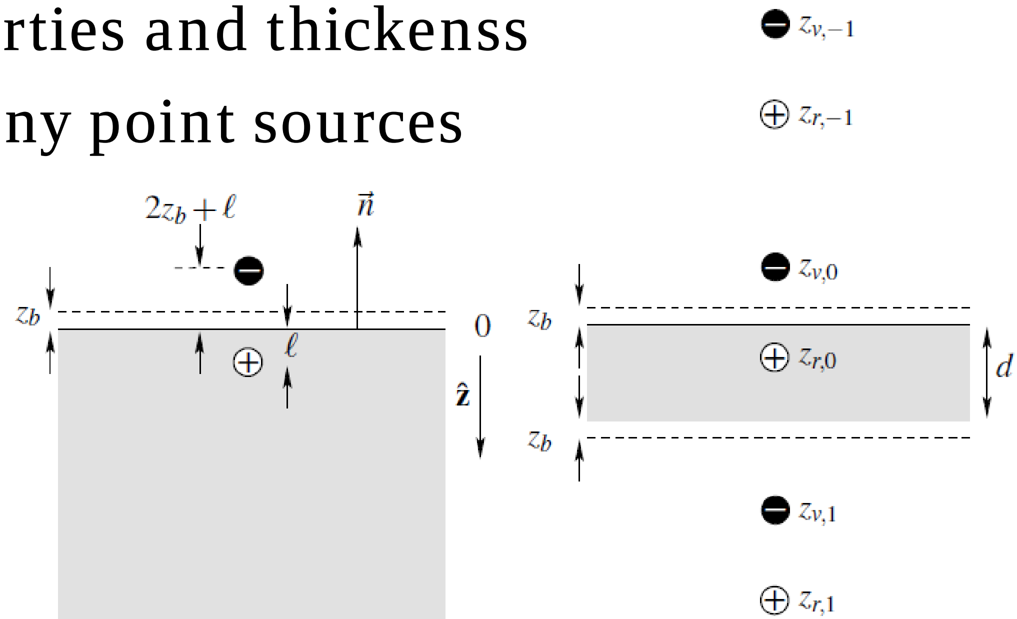
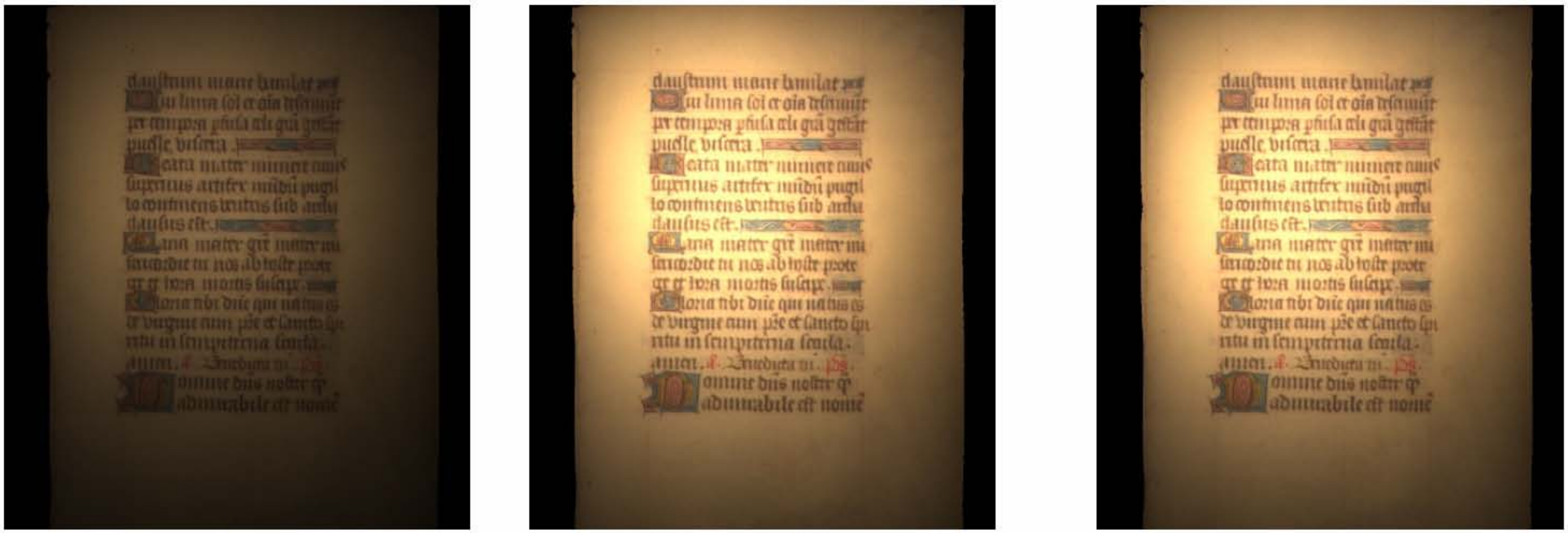


Figure 1: *Dipole configuration for semi-infinite geometry (left), and the multipole configuration for thin slabs (right).*

Diplole vs. multipole



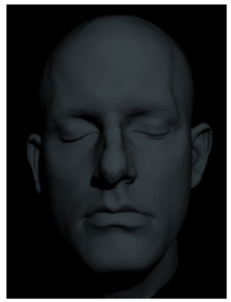
Dipole model　Multipole model　Monte Carlo reference

Figure 4: *A piece of parchment illuminated from behind. Note, how the dipole model (left) underestimates the amount of transmitted light, while the multipole model (middle) matches the reference image computed using Monte Carlo photon tracing (right).*

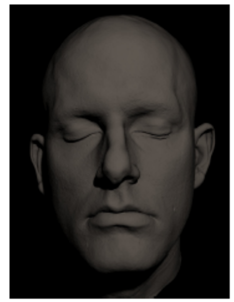
Multiple Dipole Model - Results

Epidermis Reflectance

Epidermis Transmittance



Upper Dermis Reflectance

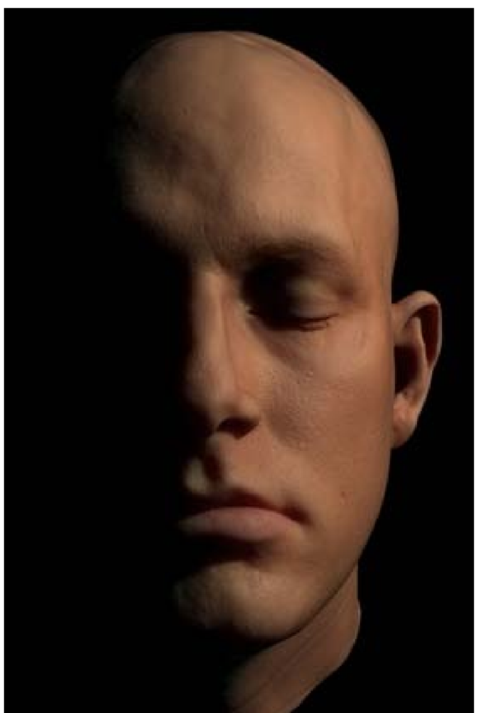


Upper Dermis Transmittance

Bloody Dermis Reflectance

Surface Roughness

All Layers



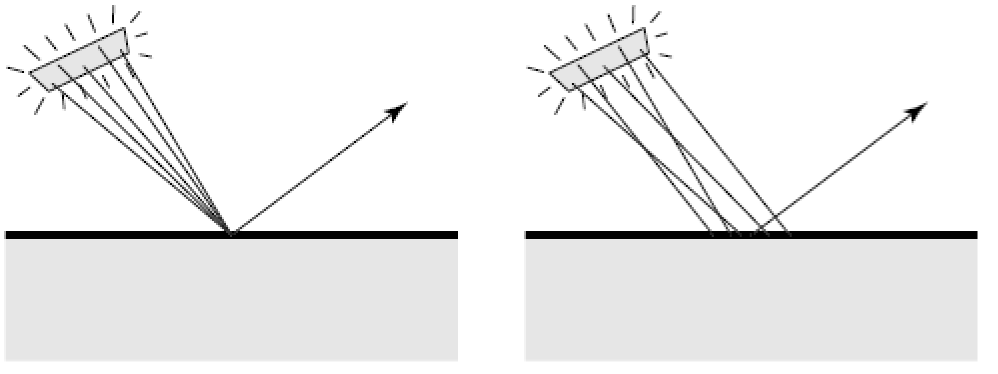
Rendering with BSSRDFs

Rendering with BSSRDFs

1. Monte Carlo sampling [Jensen et al. 2001]
2. Hierarchical method [Jensen and Buhler 2002]
3. Real-time approximations
   [d'Eon et al. 2007, Jimenez et al. 2009]

Monte Carlo sampling

- [Jensen et al. 2001]



(a) (b)

Figure 7: (a) Sampling a BRDF (traditional sampling), (b) sampling a BSSRDF (the sample points are distributed both over the surface as well as the light).

Hierarchical method

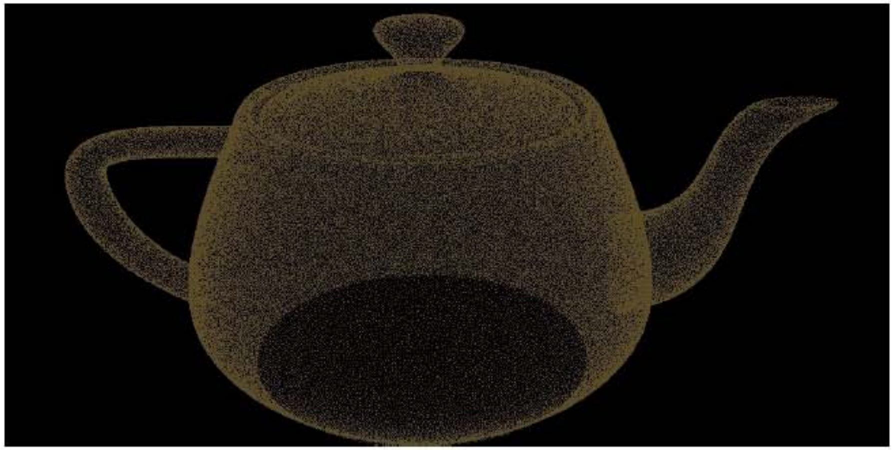
- [Jensen and Buhler 2002]
- Key idea: decouple computation of surface irradiance from integration of BSSRDF
- Algorithm
  - Distribute many points on translucent surface
  - Compute irradiance at each point
  - Build hierarchy over points (partial avg. irradiance)
  - For each visible point, integrate BSSRDF over surface using the hierarchy (far away point use higher levels)

Hierarchical method - Results

BSSRDF: sampled evaluation - 18 minutes

Illumination from a HDR environment

BSSRDF: hierarchical evaluation - 7 seconds



The sample locations on the teapot

Hierarchical method - Results

Texture-space filtering

- [d'Eon et al. 2007]
- Idea
  - Approximate diffusion profile with a sum of Gaussians
  - Blur irradiance in texture space on the GPU
    - Fast because 2D Gaussians is separable
    - Have to compensate for stretch

Diffusion profile approximation

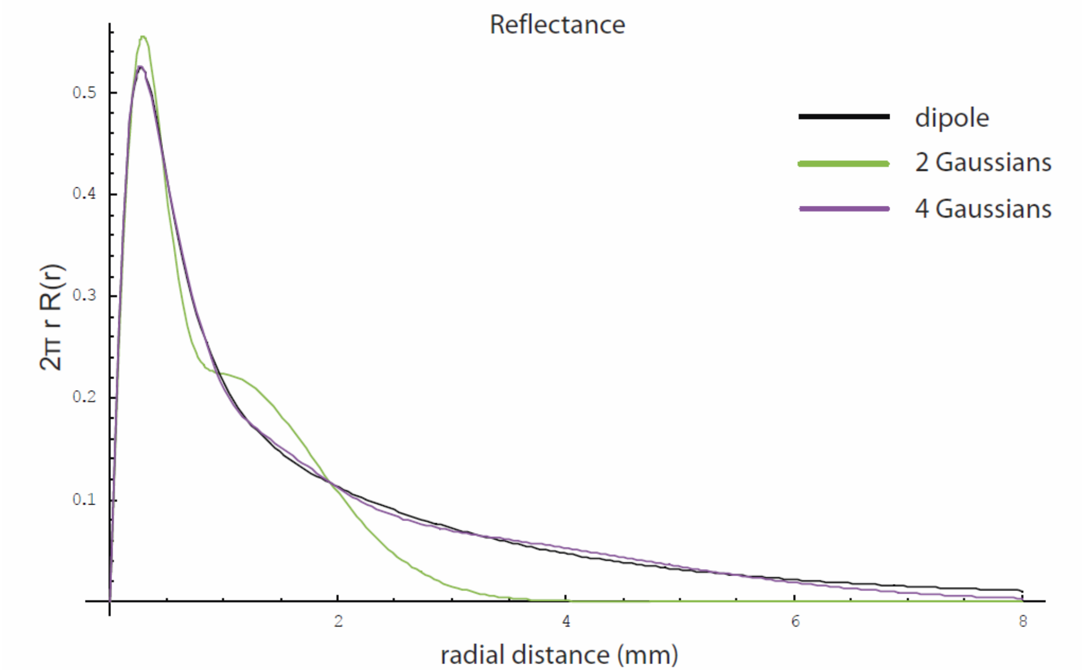


Figure 2: *Approximating a dipole with a sum of 2 and 4 Gaussians (for the green wavelength of marble [Jensen et al. 2001]). Profiles are plotted scaled by $2\pi r$, since any error in approximating the dipole is applied radially. Rendering with the 4 Gaussian sum is visually indistinguishable from using the dipole.*

Components

- Albedo (reflectance) map, i.e. texture
- Illumination
- Radiosity (=albedo * illum) filtered by the individual Gaussian kernels
- Specular reflectance

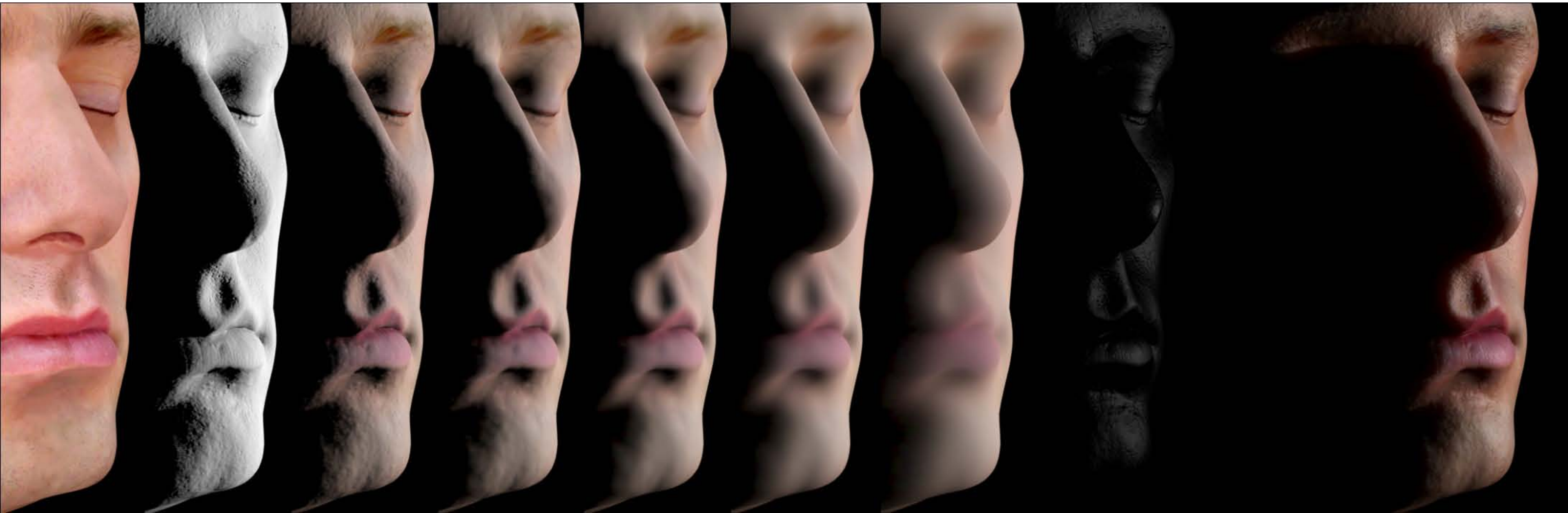


Image space filtering

- [Jimenez et al. 2009]
- Addresses scalability (think of many small characters in a game)
- Used in CryEngine 3 and other

Acquisition of scattering properties

References

- PBRT, section 16.5